The magical number 4 = 7: Span theory on capacity limitations[1]

A commentary on Cowan (2001) "The magical number 4"[2]

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Abstract: According to span theory, a behavioral theory of the magical numbers, Cowan's 4 and Miller's 7 are simply two different points on the same ogive describing the relation between performance and span load, a fundamental task characteristic. Span theory explains the magical numbers in terms of a unitary limited span ability, a mathematical abstraction from that ogive.

With this paper, cognitivist Cowan becomes something of a "strange bedfellow" in efforts to assert span theory (Bachelder & Denny, 1977a, 1977b). Cowan is right, there is a magical number, it is unitary, and developmental and individual differences are important (4.1.4); but he is wrong (4.3.1) that Miller's idea was mere rhetoric. Miller (1956) evaluated a unitary channel capacity hypothesis but his analyses failed (Bachelder, 2000). The notions of span ability and a capacity-limited focus of attention promise to fare better.

Figure 1 plots performance in the magical number tasks as a function of numbers of items in a stimulus set. The curve, an inverse ogive, suggests there is no fundamental difference between Cowan's 4 and Miller's 7; both are simply different points on the same curve.

Insert Figure1 about here

Figure 1 caption. The probability of a correct response as a function of size of stimulus set in the magical number tasks. Miller's 7 corresponds to a 50% criterion, Cowan's 4 to a 100% criterion. Pollack's data have been corrected. Mean performances on single stimuli were raised to the power of span load to estimate performance on a full stimulus set considered collectively.

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[1]This is a reproduction, prepared in September 2006, of Bachelder, B. L. (2001). The magical number 4=7: Span theory on capacity limitations. *Behavioral and Brain Sciences, 24*(1), 116-117.

[2]Cowan, N. (2001). The magical number 4 in short-term memory: A reconsideration of mental storage capacity. *Behavioral and Brain Sciences, 24*(1), 87-185.

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The similar performance limitations in these tasks are usually presumed to be coincidental. Cowan's notion of a capacity-limited focus of attention undermines that presumption (4.3.3). Section 3.3.2 extends the theory to span of apprehension (known at one time as span of attention). Section 4.4 sketches an extension to absolute judgment which would complete a unitary account.

Span theory accounts for all three limits via a mathematical proposition, an empirical generalization from data such as shown in Figure 1: In any task the probability of a correct response is an inverse ogival function of span load. Span load is the number of stimuli jointly relevant for the target response. Span ability, operationalized as the 50% threshold of the Performance X Span Load ogive, is the ability to cope with span load. This definition closely parallels Miller's definition of channel capacity as "the upper limit on the extent to which the observer can match his responses to the stimuli we give him" (p. 82).

Span theory analyses may help Cowan avoid a cul-de-sac in an extension of his theory to uni- and multidimensional absolute judgment. First, Figure 1 shows that at Cowan's 100% criterion the magical number in absolute judgment is 4, not 7. Second, the principle that increasing the number of dimensions overcomes the magical number may be an artifact of the information metric. Miller relied largely on a study by Pollack & Ficks (1954). Bachelder (1978, Part 2) translated their data from information terms back to probability of a correct response, then modeled their task as a combination of serial recall and unidimensional absolute judgment. Subjects judged the values in each of multiple dimensions then retained and reported the values as in serial recall. Mathematical models, presuming unitary magical number limitations typical for college students, generate the published data accurately (mean error = 2.7%).

Span ability is a g-like trait construct. Figure 1 presents just one of a family of parallel curves starting at the left for the developmentally young (retarded and nonretarded). Memory span and span of absolute judgment have been found to correlate .78 (60 adults: college students, average, and retarded [IQ=47.8]; summarized in Bachelder & Denny, 1977a, pp. 139-142). All three magical numbers of mildly retarded adults are smaller (5 ± 2) than those of college students (Spitz, 1973).

Cowan's concepts map to some extent onto span theory. A capacity-limited focus of attention is not that different from the notion of span ability, which can be characterized as the ability to attend to several stimuli simultaneously. Chunks usually correspond to responses. Recoding corresponds to changes in response repertoire.

Span theory tightly integrates psychometric concepts and methods into S-R style analyses of cognitive tasks. A true competition of cognitive and behavioral approaches in the analysis of the same tasks promises to enrich both traditions.

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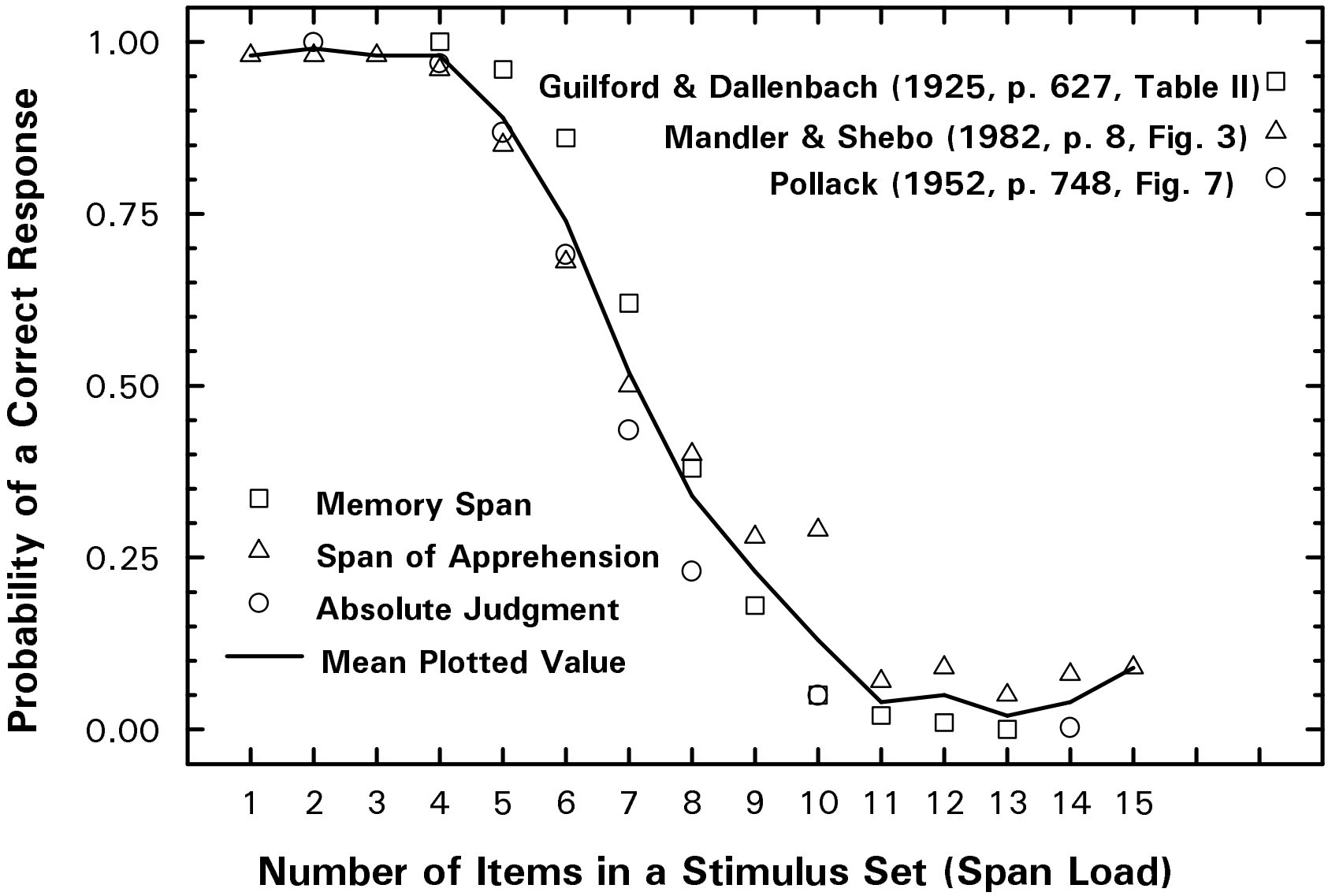
Probability of a Correct Response
1.00
0.75
0.50
0.25
0.00
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Number of Items in a Stimulus Set (Span Load)
Guilford & Dallenbach (1925, p. 627, Table II)
Mandler & Shebo (1982, p. 8, Fig. 3)
Pollack (1952, p. 748, Fig. 7)
Memory Span
Span of Apprehension
Absolute Judgment
Mean Plotted Value